



# Midas Intro Session

Simulate Concrete Cracking and  
Cracked Section Behaviors Using  
Various Approaches

Midas North American Office  
Wednesday, April 13<sup>th</sup>, 2022  
3:00 PM – 4:00 PM EDT

Presenter: JC Sun [jsun@midasoft.com](mailto:jsun@midasoft.com)  
450 7<sup>th</sup> Ave Suite 2505, New York, NY, 10123, US





## Concrete Cracking



Cracking in concrete is a complete or incomplete separation of either concrete or masonry into two or more parts produced by breaking or fracturing (ACI Concrete Terminology)

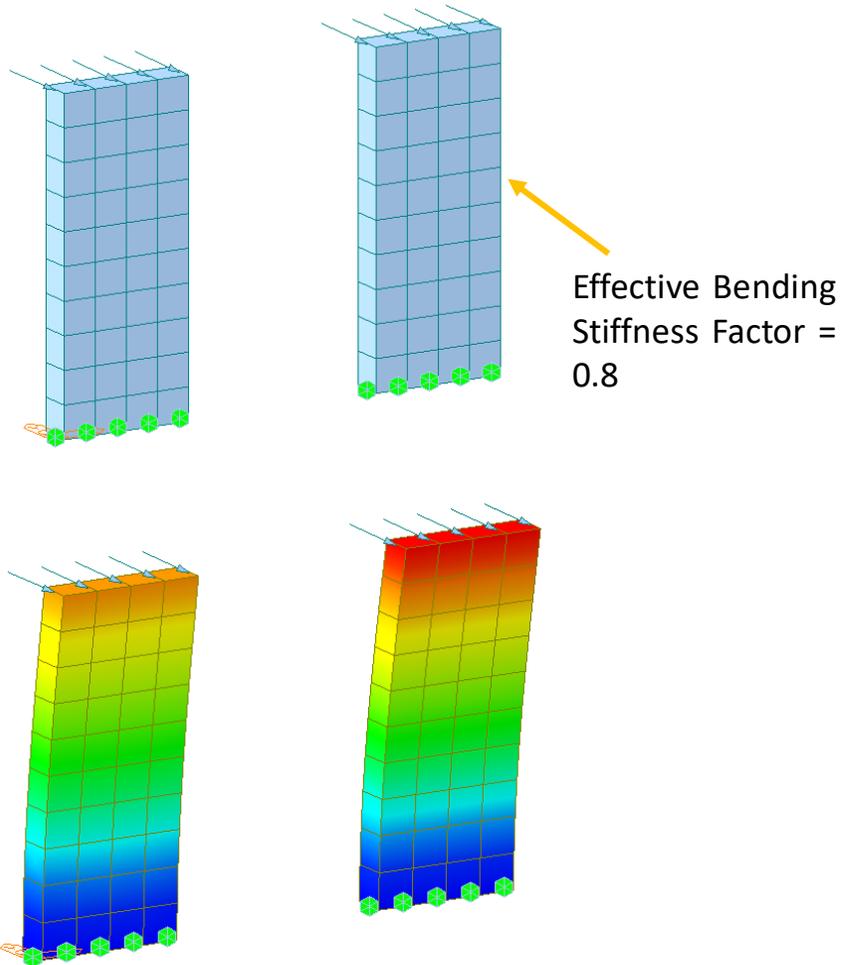
Plain Concrete is composed of coarse aggregate, sand, cement, and water. It is considered non-homogenous in micro level and homogenous in macro level.

Concrete behavior can be accessed by examining the load deformation relation. Concrete cracking and crushing occurs due to failures under excessive tensile and compressive stresses.

The behavior depends on many parameters including the composition of the concrete, the type of loading (compression / tension), the confinement effect, loading rate, temperature, etc. (Dere and Koroglu, 2017)



## Effective Flexural Rigidity



Structural models are used to determine the force and deformation demands that are used to design new structures and evaluate the performance of existing structures.

Many codes suggest carrying out elastic analysis to determine the structural behavior. However, to obtain realistic results, condition of the concrete members under loading should be reflected in the analysis.

Thus, the stiffness of the structure is represented by the effective rigidity used for all concrete members expected to be beyond the cracked state. (Avşar, et al., 2014)



## Effective Flexural Rigidity

Below some of the past researches done to simulate the effective stiffness of reinforced concrete beams and columns (Avşar, et al., 2014)

**Mirza (1990)** investigated the parameters affecting the flexural stiffness of slender columns and proposed equations for design considering the eccentricity ratio.

**Paulay and Priestley (1992)** investigated various factors affecting the flexural rigidity of RC members and proposed average values.

**Mehanny et al. (2001)** proposed simple formulas to determine effective flexural and shear coefficients of beams and columns considering the axial load level.

**Panagiotakos and Fardis (2001)** developed expressions for yield and ultimate deformation capacities of RC members that are essential for estimating the effective elastic stiffness of cracked RC members.

**Khuntia and Ghosh (2004)** proposed simple effective stiffness models to be used in the lateral analysis of frames, considering the influence of longitudinal reinforcement and eccentricity of the axial load.

**Elwood and Eberhard (2009)** proposed a 3-component model considering the effects of flexure, bar slip, and shear components of deformation that is based on the PEER Structural Performance Database.

**Kumar and Singh (2010)** proposed two different effective models for normal-strength and high-strength concrete members to be used in design of structures.

**Bonet et al. (2011)** proposed an equation to estimate the effective stiffness of slender RC columns subjected to combined axial loads and biaxial bending.



## Effective Flexural Rigidity

ACI 318R – 14 Table 6.6.3.1.1 (a) – Moment of Inertia and Cross – Sectional Area Permitted for Elastic Analysis at Factored Load Level. Taken from (MacGregor and Hage, 1977)

Member and Condition		Moment of Inertia	Cross – Sectional Area
Columns		$0.70 I_g$	$1.0 A_g$
Walls	Uncracked	$0.70 I_g$	
	Cracked	$0.35 I_g$	
Beams		$0.35 I_g$	
Flat plates and flat slabs		$0.25 I_g$	

However, the equations in Table 6.6.3.1.1 (b) provides more refined values of I considering axial load, eccentricity, reinforcement ratio, and concrete compressive strength as presented in Khuntia and Ghosh 2004 (ACI 318R - 14).



## Effective Flexural Rigidity

ACI 318R – 14 Table 6.6.3.1.1 (b) – Alternative Moments of Inertia for Elastic Analysis at Factored Load (Khuntia and Ghosh, 2004)

Member	Alternative Value of $I$ for Elastic Analysis		
	Minimum	$I$	Maximum
Columns and Walls	$0.35 I_g$	$(0.80 + 25 \frac{A_{st}}{A_g})(1 - \frac{M_u}{P_u h} - 0.5 \frac{P_u}{P_0}) I_g$	$0.875 I_g$
Beams, Flat Plates, and Flat Slabs	$0.25 I_g$	$(0.10 + 25 \rho)(1.2 - 0.2 \frac{b_w}{d}) I_g$	$0.5 I_g$

Where:

$I_g$  = moment of inertia of gross concrete section about centroid axis, neglecting reinforcement

$A_{st}$  = total area of non-prestressed longitudinal reinforcement including bars or steel shapes, excluding the prestressing reinforcement

$A_g$  = gross area of the concrete section.

$M_u$  = factored moment at section.

$P_u$  = factored axial force, compressive (+) and tensile (-)

$P_0$  = nominal axial strength at zero eccentricity.

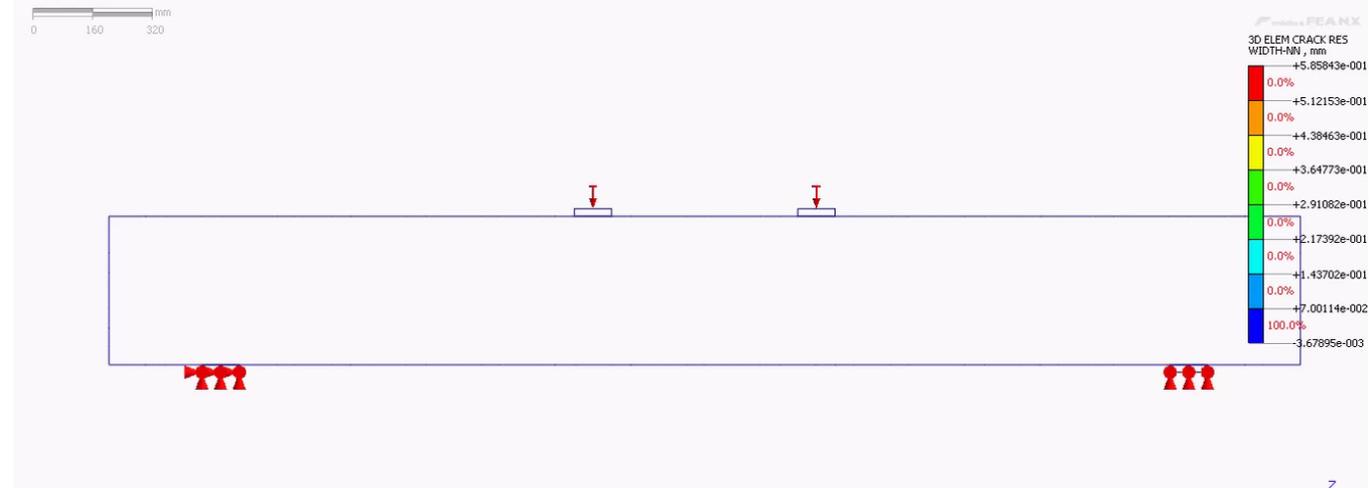
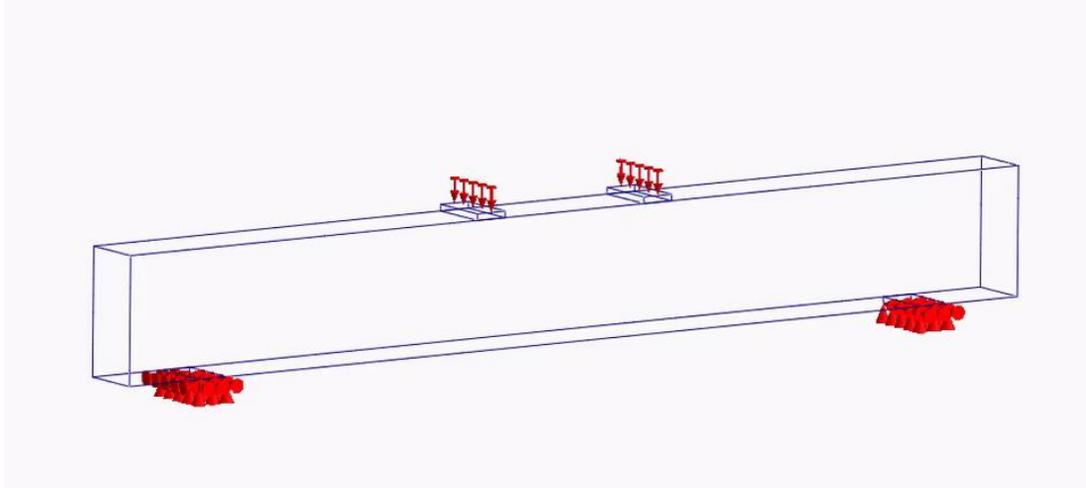
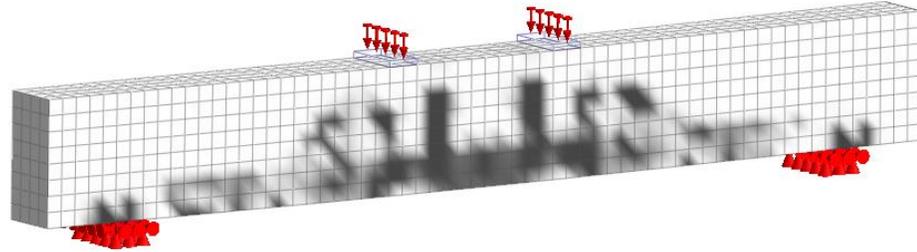
$\rho$  = ratio of  $A_s$  to  $bd$

, where  $b$  = width of compression face of member,  $d$  = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

$b_w$  = web width or diameter of circular section



# Detailed Concrete Crack Analysis





## Detailed Concrete Crack Analysis

The detailed behavior of reinforced concrete structural members under various loading and boundary conditions are often studied experimentally. The results of the tests are considered as the real behavior although many uncertainties exist in specimen production, loading, and measurement phases. Experimentally obtained responses are then compared to their analytical counterparts in order to verify if the level of errors originating from experimental uncertainties are within acceptable limits. (Dere and Koroglu, 2017)

Realistic simulations of laboratory tests of reinforced concrete specimens under monotonic and cyclic loading is quite complicated. Nonlinear constitutive relations of concrete, aggregate, interlock, tension cracks, and crushing in compression, adhesion between steel rebars and concrete cause difficulties in the modeling of reinforced concrete. (Dere, *et al.*, 2006)

Therefore, for a structural system which would be difficult to produce and test in lab settings, a full-scale and detailed simulation would help structural engineers better understand its failure mechanisms - tensile failure and compressive crushing.



- **Concrete Crack Analysis**
- **Static Analysis**
- **Fatigue Analysis**
- **Construction Stage Analysis**
- **Reinforcement Analysis**
- **Buckling Analysis**
- **Eigenvalue Analysis**
- **Response Spectrum Analysis**
- **Time History Analysis(Linear/Nonlinear)**
- **Static Contact Analysis**
- **Interface Nonlinearity Analysis**
- **Nonlinear Analysis(Material/Geometric)**
- **Heat of Hydration Analysis**
- **Heat Transfer Analysis**
- **Slope Stability Analysis**
- **Seepage Analysis**
- **Consolidation Analysis**
- **Coupled Analysis(Fully/Semi)**



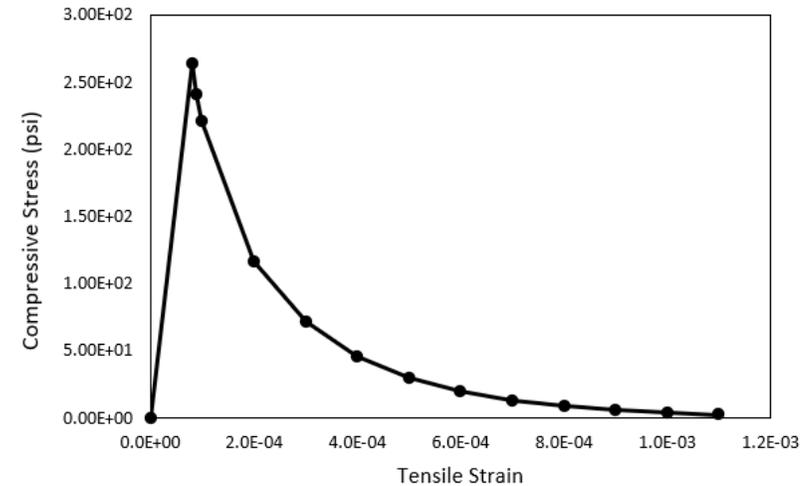
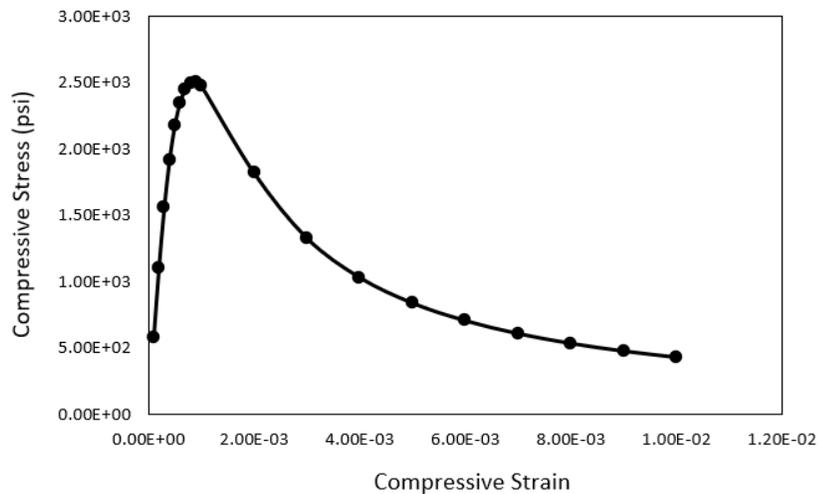
## Detailed Concrete Crack Analysis – Material Model

Two material constitutive models can be used to simulate concrete cracking

- Concrete Smeared Crack
- Concrete Damaged Plasticity

Smeared crack concrete model is preferred for applications where concrete is subjected to monotonic straining. Concrete damaged plasticity can be used with monotonic, cyclic, and/or dynamic loading conditions. (Dere and Koroglu, 2017)

To define Concrete Smeared Crack model, the concrete compressive and tensile stress/strain behaviors are needed.





## Detailed Concrete Crack Analysis – Material Model

The unconfined stress/strain relationship can be obtained based on experimental studies, however, in case when the experimental data is missing, the relationship can be approximated by the below relationship (compressive). The formulation was first proposed by (Popovics, 1973) and later modified by (Thoronfeldt et al., 1987)

For **compressive behavior**, the compressive strain ( $\varepsilon_c$ ) and stress ( $\sigma_c$ ) is given by below:

$$\frac{\sigma_c}{f'_c} = \frac{n \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)}{(n - 1) + \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^n}$$

Where  $f'_c, \varepsilon_{c0}$  are the compressive strength and strain corresponding to the maximum stress, respectively. The “n” is defined by,

$$n = 0.4 \times 10^{-3} f'_c \text{ (psi)} + 1$$

For **tensile behavior**, the tensile strain ( $\varepsilon_t$ ) and stress ( $\sigma_t$ ) is given below:

$$\sigma_t = f'_t \left( \frac{\varepsilon'_t}{\varepsilon_t} \right)^{(0.7+1000 \varepsilon_t)}, \quad \varepsilon'_t = \frac{f'_t}{E_c}$$



## References

- Dere, Y., Koroglu, M.A.** *Nonlinear FE Modeling of Reinforced Concrete*, International Journal of Structural and Civil Engineering Research Vol. 6, No. 1, February 2017.
- Avşar, Ö., Bayhan, B., Yakut, A.,** *Effective Flexural Rigidity for Ordinary Reinforced Concrete Columns and Beams*, The Structural Design of Tall and Special Buildings, November 2014.
- Dere, Y., Asgari, A., Sotelino, E.D., Archer, G.C.,** Failure Prediction of Skewed Jointed Plain Concrete Pavements Using FE Analysis, Eng. Fail. Analysis, Vol.6, 2006
- ACI 318-14** Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14). Farmington Hills, 2014.
- Propovics, S.,** A Numerical Approach to the Complete Stress-Strain Curves of Concrete, Cement and Concrete Research, Vol 3, 1973
- Thorenfeldt, E., Tomaszewics, A., Jensen, J.J.,** Mechanical Properties of High-Strength Concrete and Applications in Design, Proc. Symposium on Utilization of High-Strength Concrete, 1987