Manual calculations and software results for 2 span continuous structure

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1 Problem Statement

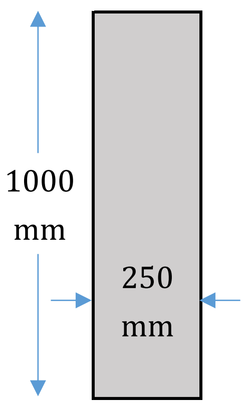


Fig. 1: Section dimension

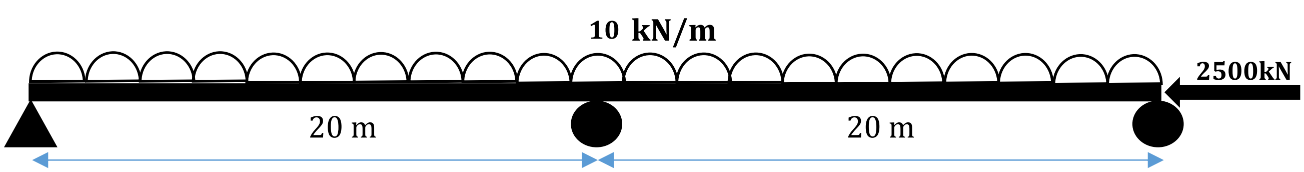
A 2 span continuous beam is resting on supports when a UDL of 10kN/m is applied on it, along with an axial load of 2500kN. After applying this load, the end supports are fixed. Calculate the primary & secondary creep effects up to 100 years. Consider time-dependet behavior in accordance to CEB-FIP 1990.

Compressive strength of concrete at 28 days:

f_{ck} = **35 MPa**

Length of each span: 20 m

Stage 1: Applied forces & boundary condition



Stage 2: New boundary condition at stage 2.
Forces applied in Stage 1 are NOT removed

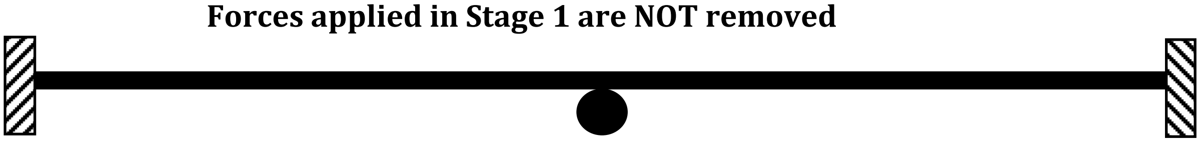


Fig. 2: Loading & stage definition

2 Section property calculations

Sectional moment of inertia (I_{yy})	2.08E+10	mm^4
C/s area of section (A_c)	250000	mm^2
Perimeter of section (u)	2500	mm

3 Modulus of elasticity

Characteristic compressive strength of concrete at 28 days (f_{ck}): 35 MPa

Modulus of elasticity at 28 days could be derived using equation 2.1-15 of CEB-FIP 1990.

$$E_{ci} = E_{co}[(f_{ck} + \Delta f)/f_{cmo}]^{1/3}$$ Eq. 2.1-15, CEB-FIP 1990

where,

E_{co}	=	21500	MPa
Δf	=	8	MPa
f_{cmo}	=	10	MPa
$\therefore E_{ci}$	=	34961.87	MPa

Since this document deals with creep only, the time-dependent behavior for change in modulus of elasticity isn't defined.

4 Creep coefficient calculation

Creep coefficient can be calculated from equation 2.1-64 of CEB-FIP 1990.

$$\emptyset(t, t_0) = \emptyset_0 \beta_c (t - t_0)$$ Eq. 2.1-64, CEB-FIP 1990

$$\emptyset_0 = \emptyset_{RH} \beta(f_{cm}) \beta(t_0)$$ Eq. 2.1-65, CEB-FIP 1990

$$\emptyset_{RH} = 1 + \frac{1 - RH/RH_0}{0.46(h/h_0)^{1/3}}$$ Eq. 2.1-66, CEB-FIP 1990

$$\beta(f_{cm}) = \frac{5.3}{(f_{cm}/f_{cmo})^{0.5}}$$ Eq. 2.1-67, CEB-FIP 1990

$$\beta(t_0) = \frac{1}{0.1 + (t_0/t_1)^{0.2}}$$ Eq. 2.1-68, CEB-FIP 1990

$$h = 2A_c/u$$ Eq. 2.1-69, CEB-FIP 1990

$$\beta_c(t-t_0) = \left[\frac{(t-t_0)/t_1}{\beta_H + (t-t_0)/t_1}\right]^{0.3}$$ Eq. 2.1-70, CEB-FIP 1990

$$\beta_H = 150\left\{1+\left(1.2\frac{RH}{RH_0}\right)^{18}\right\}\frac{h}{h_0} + 250 \le 1500$$ Eq. 2.1-71, CEB-FIP 1990

For the beam being considered in this example, the parameters are as below:

h_0 = 100 mm

h = 200 mm

RH_0 = 100 %

RH = 70 %

t_1 = 1 day

t_0 = 3 day

β_H = 563

$\beta(t_0)$ = 0.743

f_{cmo} = 10 MPa

f_{cm} = 43 MPa

$\beta(f_{cm})$ = 2.556

ϕ_{RH} = 1.518

ϕ_0 = 2.882

Manual Calculations:

Age (t) days	$\beta(t-t_0)$	Creep Coeff. $\phi(t-t_0)$	Creep Coeff. Per stage
19.69	0.344966	0.994320306	0.994320306
129.18	0.600892	1.731991381	0.737671076
847.66	0.857936	2.472886645	0.740895264
5562.35	0.971475	2.800149109	0.327262464
36500.00	0.995418	2.869161761	0.069012652

midas Civil output:

Creep Function Data Type

(•) Creep Coefficient

( ) Shrinkage Strain

Start Loading : 3 Day

End Loading : 36500 Day

Num. of Steps : 5

	Time (day)	Value
1	19.69	9.9425e-001
2	129.18	1.7320e+000
3	847.66	2.4729e+000
4	5562.35	2.8001e+000
5	36500.0	2.8692e+000
6		

midas Civil divides each construction stage into several parts. However, for easy manual verification, the creep coefficients have been directly applied in the software as calculated above. So, the calculations are done only at 5 steps. To do this, the creep coefficients as calculated above are directly applied to the elements.

5 Reaction & moment calculations

i) Dead load calculations:

Axial force in beam (Fx) = 2500 kN

Mid span deflection of a simply supported beam subjected to UDL can be obtained using equation:

$$\delta = \frac{5wL^4}{384EI} = 0.457641 \text{ m}$$

Reaction at mid support will be the point load that makes this deflection 0.

$$\delta = \frac{PL^3}{48EI} \qquad \therefore \; P = \frac{48EI\delta}{L^3}$$

$$\therefore \; R_2 = \frac{48 \times 34961866.63 \times 2.08333E\text{-}02 \times 0.457641469}{40^{\,3}}$$

$$= 250 \text{ kN}$$

$$R_1 = R_3 = 75 \text{ kN}$$

The forces & deformations for the 2 span continuous beam are tabulated on the next page. These will be used further in creep deformation & force calculations.

The creep calculations in the table are for stage 1 values.

Manual calculations and software results for 2 span continuous structure

Distance from left	DL moment	Creep Pr. Moment	DL Cumm. axial deform.	Creep Cum. axial deform.
(m)	(kNm)	(kNm)	(mm)	(mm)
0	0	0.00	0	0
1	70	69.60	0.286025918	0.284401378
2	130	129.26	0.572051836	0.568802757
3	180	178.98	0.858077754	0.853204135
4	220	218.75	1.144103672	1.137605513
5	250	248.58	1.43012959	1.422006891
6	270	268.47	1.716155508	1.70640827
7	280	278.41	2.002181426	1.990809648
8	280	278.41	2.288207344	2.275211026
9	270	268.47	2.574233262	2.559612405
10	250	248.58	2.860259181	2.844013783
11	220	218.75	3.146285099	3.128415161
12	180	178.98	3.432311017	3.41281654
13	130	129.26	3.718336935	3.697217918
14	70	69.60	4.004362853	3.981619296
15	0	0.00	4.290388771	4.266020674
16	-80	-79.55	4.576414689	4.550422053
17	-170	-169.03	4.862440607	4.834823431
18	-270	-268.47	5.148466525	5.119224809
19	-380	-377.84	5.434492443	5.403626188
20	-500	-497.16	5.720518361	5.688027566
21	-380	-377.84	6.006544279	5.972428944
22	-270	-268.47	6.292570197	6.256830323
23	-170	-169.03	6.578596115	6.541231701
24	-80	-79.55	6.864622033	6.825633079
25	0	0.00	7.150647951	7.110034457
26	70	69.60	7.436673869	7.394435836
27	130	129.26	7.722699787	7.678837214
28	180	178.98	8.008725705	7.963238592

Distance from left	DL moment	Creep Pr. Moment	DL Cumm. axial deform.	Creep Cum. axial deform.
(m)	(kNm)	(kNm)	(mm)	(mm)
29	220	218.75	8.294751624	8.247639971
30	250	248.58	8.580777542	8.532041349
31	270	268.47	8.86680346	8.816442727
32	280	278.41	9.152829378	9.100844105
33	280	278.41	9.438855296	9.385245484
34	270	268.47	9.724881214	9.669646862
35	250	248.58	10.01090713	9.95404824
36	220	218.75	10.29693305	10.23844962
37	180	178.98	10.58295897	10.522851
38	130	129.26	10.86898489	10.80725238
39	70	69.60	11.1550108	11.09165375
40	0	0.00	11.44103672	11.37605513

Stage 2:

In stage 2, the boundary condition at both ends is changed to fix. However, the creep primary effect would be in accordance to support condition at the time of applying the load. Hence, primary effect would continue in accordance to support condition in stage 1. However, support condition of stage 2 would give rise to the secondary effects. Reactions for this could be calculated as below.

Step 1: Obtain the magnitude of UDL for simply support beam, that would cause bending moments as obtain in table above for additional creep primary.

Equivalent UDL = Creep coefficient for stage 2 x applied UDL in stage 1
= 0.737671076 x 10
= 7.376710755 kN/m

Step 2: Apply this equivalent UDL on beam as in stage 2 & get support reactions. For a beam with fixed end supports and no intermediate support, the maximum deflection at mid of the beam could be obtained as below.

$$\delta = \frac{wL^4}{384EI} = 67.51777 \text{ mm}$$

Reaction at mid support will be the point load that makes this deflection 0.

$$\delta = \frac{PL^3}{192EI} \quad \therefore P = \frac{192EI\delta}{L^3}$$

$$\therefore R_2 = \frac{192 \times 34961.87 \times 2.08333E+10 \times 67.51777}{40000^3}$$

$$= 147.53 \text{ kN} \qquad R_1 = R_3 = 73.76711 \text{ kN}$$

<u>Step 3:</u> Apply this equivalent UDL on beam as in stage 1 & get support reactions.

$$\delta = \frac{5wL^4}{384EI} = 337.59 \text{ mm}$$

Reaction at mid support will be the point load that makes this deflection 0.

$$\delta = \frac{PL^3}{48EI} \quad \therefore P = \frac{48EI\delta}{L^3}$$

$$\therefore R_2 = \frac{48 \times 34961.87 \times 2.08333E+10 \times 337.58887}{40000^3}$$

$$= 184.42 \text{ kN} \qquad R_1 = R_3 = 55.32533 \text{ kN}$$

<u>Step 4:</u> Obtain creep reaction and support moments

Creep reaction would be difference between reactions of simply supported and continuous condition of the beam.

$\therefore$ Creep Reaction = Step 2 reaction - Step 3 reaction

$\therefore R_2 = 147.53 - 184.42 = -36.88$ kN

$\therefore R_1 = R_3 = 73.77 - 55.33 = 18.44$ kN

Since beam is symmetric, rotation at internal support is 0 under constant UDL.

$\therefore$ Support moment would be same at all the support location (As for fixed).

$\therefore M_{Sup} = wL^2/12 = -245.89$ kNm

$M_{mid} = wL^2/24 = 122.9452$ kNm

Moment at distance x from the support could be obtained using the formula:

$$M_x = w\,(6Lx - L^2 - 6x^2)/12$$

Creep sec. moment = Creep moment in fixed beam - Additional Creep Pr. in SS beam

Manual calculations and software results for 2 span continuous structure

Dist. from left	Creep Pr. Moment		Creep Sec. Moment
	Total	Additional	
(m)	(kNm)	(kNm)	(kNm)
0	0.00	0.00	-245.89
1	121.24	51.64	-227.45
2	225.16	95.90	-209.01
3	311.76	132.78	-190.57
4	381.04	162.29	-172.12
5	433.00	184.42	-153.68
6	467.64	199.17	-135.24
7	484.96	206.55	-116.80
8	484.96	206.55	-98.36
9	467.64	199.17	-79.91
10	433.00	184.42	-61.47
11	381.04	162.29	-43.03
12	311.76	132.78	-24.59
13	225.16	95.90	-6.15
14	121.24	51.64	12.29
15	0.00	0.00	30.74
16	-138.56	-59.01	49.18
17	-294.44	-125.40	67.62
18	-467.64	-199.17	86.06
19	-658.16	-280.32	104.50
20	-866.00	-368.84	122.95
21	-658.16	-280.32	104.50
22	-467.64	-199.17	86.06
23	-294.44	-125.40	67.62
24	-138.56	-59.01	49.18
25	0.00	0.00	30.74
26	121.24	51.64	12.29
27	225.16	95.90	-6.15
28	311.76	132.78	-24.59

Dist. from left	Creep Pr. Moment		Creep Sec. Moment
	Total	Additional	
(m)	(kNm)	(kNm)	(kNm)
29	381.04	162.29	-43.03
30	433.00	184.42	-61.47
31	467.64	199.17	-79.91
32	484.96	206.55	-98.36
33	484.96	206.55	-116.80
34	467.64	199.17	-135.24
35	433.00	184.42	-153.68
36	381.04	162.29	-172.12
37	311.76	132.78	-190.57
38	225.16	95.90	-209.01
39	121.24	51.64	-227.45
40	0.00	0.00	-245.89

Axial reaction due to creep in stage 2 woule be obtained very simply by multiplying creep coefficient with the applied axial force in stage 1.

$\therefore$ FX = 0.737671 x 2500

= 1844.178 kN